

Solutions

(a) (i) $y = x + 3$

$$\frac{dy}{dx} = 1$$

[2]

(ii) $y = 4x^3 + 10x^{1/2}$

$$\begin{aligned}\frac{dy}{dx} &= 12x^2 + 5x^{-1/2} \\ &= 12x^2 + \frac{5}{\sqrt{x}}\end{aligned}$$

[3]

(iii) $y = \frac{1}{x^5} + x^5$

$$y = x^{-5} + x^5$$

$$\begin{aligned}\frac{dy}{dx} &= -5x^{-6} + 5x^4 \\ &= \frac{-5}{x^6} + 5x^4\end{aligned}$$

[3]

(iv) $y = \sin(8x)$

$$\frac{dy}{dx} = 8\cos(8x)$$

[2]

$$v) \quad y = x \cdot \cos(x)$$

$$\frac{dy}{dx} = 1 \cdot \cos(x) + x(-\sin(x))$$

$$= \cos(x) - x(\sin(x))$$

(Product Rule)

[5]

$$(vi) \quad y = (2x-3)^{10}$$

$$\frac{dy}{dx} = 10(2x-3)^9 \cdot 2$$

$$= 20(2x-3)^9$$

(Chain Rule)

[3]

$$(vii) \quad y = 3x + \tan(x)$$

$$\frac{dy}{dx} = 3 + \sec^2(x)$$

[2]

$$\underline{1b} \quad y = \frac{1}{x} + \cos(2x)$$

$$\Rightarrow y = x^{-1} + \cos(2x)$$

$$\frac{dy}{dx} = -1x^{-2} - 2\sin(2x)$$

$$\frac{d^2y}{dx^2} = +2x^{-3} - 4\cos(2x)$$

$$\frac{d^3y}{dx^3} = -6x^{-4} + 8\sin(2x) \quad [4]$$

$$\underline{1c} \quad y = \frac{(x-1) \leftarrow u}{(4+x^2) \leftarrow v}$$

$$\frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2} \quad (\text{Quotient Rule})$$

$$= \frac{(4+x^2) \cdot 1 - (x-1) \cdot 2x}{(4+x^2)^2}$$

$$= \frac{4+x^2-2x^2+2x}{(4+x^2)^2}$$

$$= \frac{4+2x-x^2}{(4+x^2)^2}$$

$$\therefore \text{ when } x=0, \quad \frac{dy}{dx} = \frac{4}{(4)^2}$$

$$\frac{dy}{dx} = \frac{1}{4}$$

[6]

$$\begin{aligned}
 \text{Q2a) (i)} \quad & \int (10x + x^3) dx \\
 &= \frac{10x^2}{2} + \frac{x^4}{4} + C \\
 &= \underline{5x^2 + \frac{x^4}{4} + C} \quad [3]
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \int (x - 3e^x) dx \\
 &= \underline{\frac{x^2}{2} - 3e^x + C} \quad [3]
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & \int \left(\frac{1}{x} + 4\right) dx \\
 &= \underline{\ln x + 4x + C} \quad [3]
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & \int 2x(3x + 4) dx \\
 &= \int (6x^2 + 8x) dx \\
 &= \frac{6x^3}{3} + \frac{8x^2}{2} + C \\
 &= \underline{2x^3 + 4x^2 + C} \quad [3]
 \end{aligned}$$

$$\begin{aligned}
 \text{v)} \quad & \int_0^2 (3x^2 + 2x) dx \\
 &= \left[\frac{3x^3}{3} + \frac{2x^2}{2} \right]_0^2 \\
 &= [x^3 + x^2]_0^2 \\
 &= (2)^3 + (2)^2 \\
 &= 8 + 4 \\
 &= \underline{12}
 \end{aligned}
 \quad [3]$$

2b let $I = \int_0^1 \frac{x dx}{\sqrt{x^2 + 1}}$

let $u = x^2 + 1$
 when $x=0$, $u=1$
 $x=1$, $u=(1)^2 + 1 = 2$

$$\begin{aligned}
 \frac{du}{dx} &= 2x \\
 \Rightarrow \frac{du}{2} &= x dx
 \end{aligned}$$

$$\therefore I = \int_1^2 \frac{\frac{du}{2}}{\sqrt{u}}$$

$$\therefore I = \frac{1}{2} \int_1^2 \frac{du}{\sqrt{u}}$$

$$I = \frac{1}{2} \int_1^2 u^{-1/2} du$$

$$= \frac{1}{2} \cdot \left[\frac{u^{1/2}}{1/2} \right]_1^2$$

$$= \frac{1}{2} \cdot [2\sqrt{u}]_1^2$$

$$= \frac{1}{2} \cdot \{2\sqrt{2} - 2\sqrt{1}\}$$

$$= \frac{1}{2} \cdot \cancel{2} \cdot \{\sqrt{2} - 1\}$$

$$= \sqrt{2} - 1$$

$$= \underline{0.4142 \text{ (4 d.p's)}}$$

[5]

$$2c) i) \quad I = \int x \cdot \sin(2x) dx$$

$$= \frac{-\cos(2x) \cdot x}{2} - \int \frac{-\cos(2x)}{2} dx$$

$$= -\frac{x \cos(2x)}{2} + \frac{1}{2} \int \cos(2x) dx$$

$$= -\frac{x \cos(2x)}{2} + \frac{1}{2} \cdot \frac{\sin(2x)}{2} + C$$

$$= -\frac{x \cos(2x)}{2} + \frac{\sin(2x)}{4} + C$$

(Integration by PARTS) [5]

$$ii) \quad I = \int_1^e x \cdot \ln(x) dx.$$

$$= \left[\frac{x^2}{2} \cdot \ln(x) \right]_1^e - \int_1^e \frac{x^2}{2} \cdot \frac{1}{x} dx$$

$$= \frac{e^2}{2} \ln(e) - \frac{1^2}{2} \ln(1) - \frac{1}{2} \int_1^e x dx$$

$$= \frac{e^2}{2} - 0 - \frac{1}{2} \cdot \left[\frac{x^2}{2} \right]_1^e$$

$$= \frac{e^2}{2} - \frac{1}{2} \left(\frac{e^2}{2} - \frac{1}{2} \right)$$

$$= \underline{\underline{\frac{e^2}{4} + \frac{1}{4}}}$$

[5]
(Int. by PARTS)

Q3

(i) let $z = (3+2i)(1-i)$

$$= 3 + 2i - 3i - 2i^2$$

$$= 3 - i - 2(-1)$$

$$= 5 - i$$

[2]

(ii) let $z = \frac{3}{(2+i)}$

$$z = \frac{3}{(2+i)} \frac{(2-i)}{(2-i)}$$

$$z = \frac{3(2-i)}{4 + \cancel{2i} - \cancel{2i} - i^2}$$

$$z = \frac{3(2-i)}{5}$$

$$z = \frac{6}{5} - \frac{3}{5}i$$

[2]

$$(iii) \text{ let } z = \frac{4}{i^5}$$

$$z = \frac{4}{i^2 \cdot i^2 \cdot i}$$

$$z = \frac{4}{(-1)(-1) \cdot i}$$

$$z = \frac{4}{i}$$

$$z = \frac{4 \cdot i}{i \cdot i}$$

$$z = \frac{4i}{-1}$$

$$\Rightarrow \underline{z = 0 - 4i}$$

[2]

$$b) \quad z_1 = \frac{(2-j)}{(2+j)}$$

$$z_1 = \frac{(2-j)(2-j)}{(2+j)(2-j)}$$

$$z_1 = \frac{4 - 4j + j^2}{4 + 2j - 2j - j^2}$$

$$\therefore z_1 = \frac{3-4j}{5}$$

$$z_1 = \frac{3}{5} - \frac{4}{5}j \quad [3]$$

$$z_2 = \frac{(3+j)}{(3-j)}$$

$$= \frac{(3+j)(3+j)}{(3-j)(3+j)}$$

$$= \frac{9+3j+3j+j^2}{9-\cancel{3j}+\cancel{3j}-j^2}$$

$$z_2 = \frac{8+6j}{10}$$

$$= \frac{\cancel{2}(4+3j)}{\cancel{10}5}$$

$$z_2 = \frac{4}{5} + \frac{3}{5}j \quad [3]$$

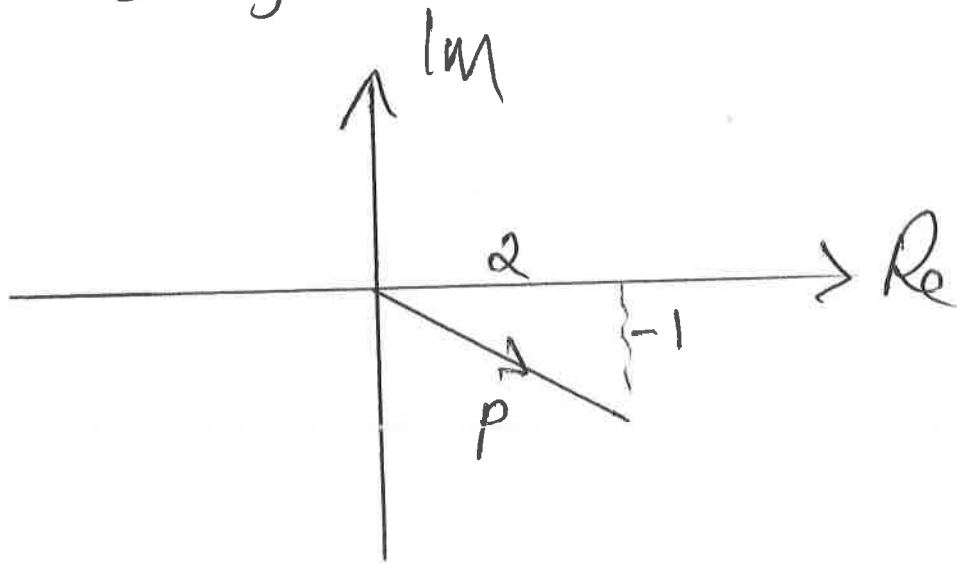
$$C(i) \quad p = 2z_1 + z_2$$

$$= 2 \left[\frac{3}{5} - \frac{4}{5}j \right] + \left[\frac{4}{5} + \frac{3}{5}j \right]$$

$$= \frac{6}{5} - \frac{8}{5}j + \frac{4}{5} + \frac{3}{5}j$$

$$= \frac{10}{5} - \frac{5}{5}j$$

$$p = 2 - j$$



(ii)

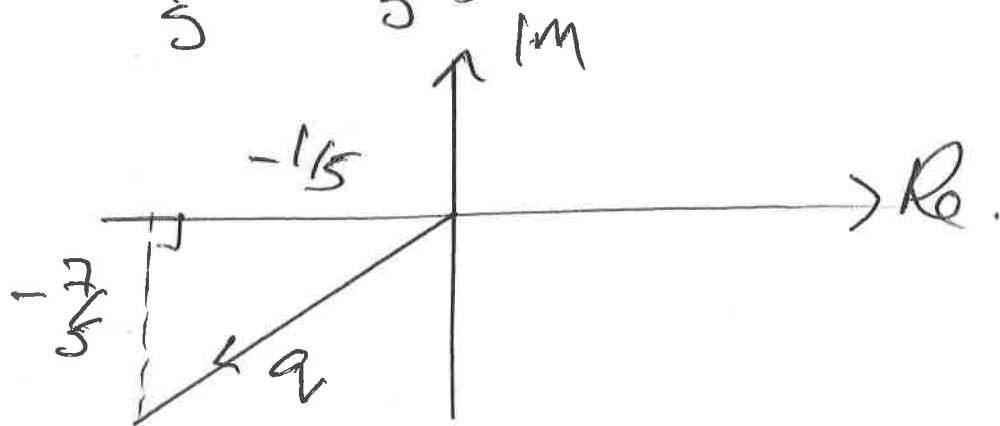
$$q = z_1 - z_2$$

$$= \left[\frac{3}{5} - \frac{4}{5}j \right] - \left[\frac{4}{5} + \frac{3}{5}j \right]$$

$$= \frac{3}{5} - \frac{4}{5}j - \frac{4}{5} - \frac{3}{5}j$$

$$q = -\frac{1}{5} - \frac{7}{5}j$$

$$q = -\frac{1}{5} - \frac{7}{5}j$$



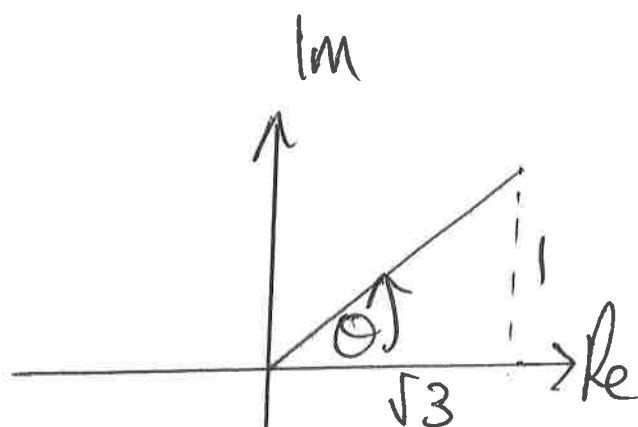
3d $z = \sqrt{3} + i$

$$|z| = \sqrt{(\sqrt{3})^2 + (1)^2}$$

$$|z| = \sqrt{3 + 1}$$

$$|z| = \sqrt{4}$$

$$\underline{|z| = 2.}$$



$$\text{Arg}(z) = \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \tan^{-1}(1/\sqrt{3})$$

$$\underline{\theta = 30^\circ.}$$

$$\therefore z = r [\cos \theta + i \sin \theta]$$

$$\underline{z = 2 [\cos(30^\circ) + i \sin(30^\circ)]}$$

[6]

e) let $z = \frac{1+i}{1-i}$

$$z = \frac{(1+i)(1+i)}{(1-i)(1+i)}$$

$$z = \frac{1+i+i+i^2}{1-\cancel{i}+\cancel{i}-i^2}$$

$$z = \frac{\cancel{1}+2i-\cancel{1}}{2}$$

$$z = \frac{2i}{2}$$

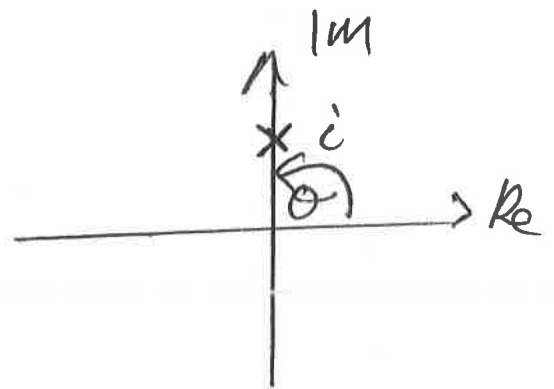
$$\underline{z = i}$$

$$|z| = 1 = r \quad \text{Arg}(z) = 90^\circ$$

$$z = r [\cos \theta + i \sin \theta]$$

$$\underline{z = 1 [\cos(90^\circ) + i \sin(90^\circ)]}$$

[6]



Q4a i) $3^{2x} = 2^{x+1}$
 $\log_{10} 3^{2x} = \log_{10} 2^{x+1}$

$$2x \log_{10} 3 = (x+1) \log_{10} 2$$

$$2x \log_{10} 3 = x \log_{10} 2 + \log_{10} 2$$

$$2x \log_{10} 3 - x \log_{10} 2 = \log_{10} 2$$

$$x [2 \log_{10} 3 - \log_{10} 2] = \log_{10} 2$$

$$x = \frac{\log_{10} 2}{[2 \log_{10} 3 - \log_{10} 2]}$$

$$= \underline{\hspace{2cm}} \text{ (2dp's) [4]}$$

(ii) $2 \ln x = \ln(5x-6)$

$$\Rightarrow \ln x^2 = \ln(5x-6)$$

$$\Rightarrow x^2 = 5x-6$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$\Rightarrow \underline{\text{either}} \quad x = 2$$

$$\underline{\text{or}} \quad x = 3$$

[2]

4b (i) $\log(x^6 \cdot y \cdot z)$

$$= \log x^6 + \log y + \log z$$

$$= \underline{6 \log x + \log y + \log z} \quad [2]$$

(ii) $\ln \left[\frac{x^3 y}{z} \right] = \ln [x^3 y] - \ln(z)$

$$= \ln x^3 + \ln y - \ln(z)$$

$$= \underline{3 \ln x + \ln y - \ln(z)} \quad [2]$$

